

Fig. 8. Axes of the rigid body vibration.

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The Crystal Structure of Nb₂₁S₈*

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The crystal structure of Nb₂₁S₈ has been determined from single-crystal X-ray diffraction data. The compound crystallizes in the tetragonal system, space group *I4/m* with unit-cell parameters $a = 16.794 \text{ \AA}$, $c = 3.359 \text{ \AA}$. The unit cell contains two formula units. All atoms are located in layers separated by $c/2$. Refinement of the structure was performed by least-squares treatment of x and y positional parameters and isotropic temperature factor coefficients. Pauling bond orders indicate that the coordination of sulfur in this compound is seven. This result is consistent with sulfur atoms utilizing d orbitals to form electron deficient bonds to the metal atoms.

Introduction

The high temperature (*ca.* 1350°C) preparation of a new subsulfide in the niobium-sulfur system, characterized as Nb₂S, and some crystallographic data for the compound were recently reported (Franzen, DeJong & Conard, 1966). The compositions reported were obtained by combustion techniques on samples which have since been identified as containing two phases. A more recent composition analysis is reported below.

Previous studies of the niobium-sulfur system in the range $0 \leq S/Nb \leq 1.0$ resulted in the identification of two phases, neither of which was the same as the phase reported here. Biltz & Köcher (1938) reported a phase with the homogeneity range $0.5 \leq S/Nb \leq 1.0$; Jellinek

(1962) observed a NbS_{1-x} ($x = 0.1$ to ?) phase with an ordered NiAs-type structure.

Investigations of the lower sulfides of Hf and Ti, namely Hf₂S (Franzen & Graham, 1966) and Ti₂S (Owens, Conard & Franzen, 1966), have resulted in a new interpretation of the bonding of sulfur in the mono-sulfides and lower sulfides of transition metals (Franzen, 1966). The interest in the crystal structure of Nb₂₁S₈ stemmed from a desire to describe more completely the chemistry of sulfur in the lower sulfides of transition metals.

Experimental

The previously reported crystallographic data for Nb₂₁S₈ are summarized as follows. The space group was reported to be *I4/m*, the lattice parameters reported were $a = 16.794 \pm 0.005$, $c = 3.359 \pm 0.002 \text{ \AA}$, and the number of 'Nb₂S' formula units per unit cell was estimated to be between 21 and 24, based on a measured

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density of 8.8 g.cm^{-3} (which corresponded to 2.3 Nb_{21}S_8 formula units per cell).

The density has been redetermined for a single phase sample of Nb_{21}S_8 to be $7.8 \pm 0.1 \text{ g.cm}^{-3}$, which corresponds to 2.0 Nb_{21}S_8 formula units per unit cell. More recent combustion analyses have been performed on single-phase samples under more carefully controlled conditions than before and yielded $\text{S/Nb} = 0.34$.

Intensity data were collected with Mo $K\alpha$ radiation using a General Electric spectrogoniometer with a scintillation counter detector. A total of 557 independent reflections of $hk0$ and $hk1$ types were measured. Lorentz and polarization corrections were applied. Absorption corrections, based on the crystal's trapezoidal shape (average side length of about 40μ and thickness of about 30μ) were made with use of a program by Busing & Levy (1957) adapted to the IBM 360/50 computer. A total linear absorption coefficient of 136.5 cm^{-1} was used in the absorption correction calculation. No extinction corrections were made.

Structure determination

Unitary structure factors were obtained from the observed structure factors by a graphical method (Woolfson, 1961). Except for attenuation normally expected for the increase in scattering angle, the intensity data were observed to depend on l only so far as l was even or odd, indicating a layered structure with all atoms in the mirror planes of the centrosymmetric space group (Franzen, DeJong & Conard, 1966). It follows that $U_{hkl+2} = U_{hkl} = U_{\bar{k}hl} = U_{k\bar{h}l}$. Thus, by the Harker-Kasper inequalities (Harker & Kasper, 1948) if U_{hkl} and $U_{h-k, h+k, 0}$ are sufficiently large, then $U_{h-k, h+k, 0}$ is positive, and a similar remark holds for $U_{h+k, k-h, 0}$.

The 51 unitary structure factors with magnitudes greater than or equal to 0.39, about 9% of the total number of the observed reflections, were examined. Application of the reasoning described above yielded the sign of 19 of these as positive. One of the two degrees of freedom with respect to sign determination for the tetragonal case is removed by the body-center (Hauptmann & Karle, 1953), and thus only one unitary structure factor sign could be arbitrarily determined. $U_{3,4,1}$ was arbitrarily taken to be positive. This sign, together with the 19 positive unitary structure factors discussed above and the Harker-Kasper inequalities, yielded 15 more positive structure factors. The sign of $U_{2,5,1}$ was designated a , and combination of this structure factor and those known to be positive with those of the original 51 whose signs were as yet undetermined yielded 13 structure factors with the sign a .

Two electron density maps were computed utilizing the 47 unitary structure factors, one map with a positive, the other with a negative. These maps contained more peaks than could be explained on the basis of the cell content given by the density and lattice parameters by about a factor of two. Of several attempts to solve the

structure utilizing these electron density maps and the Patterson map, the following scheme resulted in a structure which refined to an acceptable reliability index.

The strongest peak on the electron density map obtained with a negative was taken to be an atom position $x', y', 0$. A corresponding Patterson peak occurred at $2x', 2y', 0$ in the zero section of the Patterson map. A superposition of zero section Patterson maps was performed with the origin of the superimposed map placed at $x', y', 0$ on the primary map. A trial structure in the $x, y, 0$ plane was obtained. This trial structure was superimposed on the electron density map discussed above and a second trial structure was obtained from the coincident peaks. The second trial structure contained an atom at the origin and six other independent atom positions.

At this point in the structure determination, use was made of the known structural chemistry of lower transition metal sulfides (Owens, Conard & Franzen, 1967), and the niobium and sulfur atoms were distributed among the six independent positions and the position at the origin in an attempt to obtain a structure in which there were six niobium atoms about each sulfur atom in a trigonal prism. There was an obvious hole in the structure which resulted from this distribution, and the placement of a niobium atom at this position completed the trigonal prism of niobium atoms about one of the independent sulfur positions. This resulted in a final trial structure with a niobium in the twofold position at the origin, and five independent niobium atoms and two independent sulfur atoms in the eightfold positions, $x, y, 0$. This final structure was refined by least-squares computation (Busing, Martin & Levy, 1962) using atomic scattering factor tables given by Hansen, Herman, Lea & Skillman (1964), and after three cycles of refinement, a reliability index, defined by $R = \sum ||F_o| - |F_c|| / \sum |F_o|$, of 0.12 was obtained.

The structure was further refined by least-squares computation of the 14 positional parameters and eight isotropic temperature factor coefficients, using a weighting scheme based on statistical counting errors. The weights were assigned by calculating a relative error in the observed structure factor for each reflection, namely,

$$\sigma F = \frac{\sigma I}{2F(Lp)},$$

where σ_I is the estimated error in the intensity corrected for absorption, F is the observed structure factor and Lp is the Lorentz and polarization correction factor. The square of the error in intensity, σ_I^2 , was taken to be the total counts plus the background counts plus the square of 5% of the total counts plus the square of 5% of the background counts plus an estimated error in the absorption correction. The goodness of fit, defined by

$$\frac{\sum_i w^2(F_o - F_c)^2}{n-s}$$

was equal to 1.35, where w is the weight ($1/\sigma_F$) of the i th reflection, n is the number of reflections and s is the number of variables. A final unweighted $R=0.063$ was obtained. Table 1 lists the values obtained. Table 2 lists the observed and calculated structure factors for all 557 observed reflections, and in addition, lists 60 unobserved reflections using parentheses to indicate F_{\min} calculated using the background intensity. The unweighted R including the unobserved reflections was 0.086. Only the observed reflections were used in the least-squares refinement.

All 47 signs assigned by the direct method agreed with those obtained for the refined structure. For the purpose of checking the correctness of the refinement, a difference Fourier synthesis was performed using all 557 reflections. The synthesis showed the highest peaks, corresponding to about 0.8 electron, in the $z=0$ section. Specifically, the atomic position for Nb(2) had a positive $\rho_o - \rho_c$ corresponding to 0.8 electron, while those of Nb(3) and Nb(5) had negative peaks

corresponding to about 0.7 electron. All other atomic positions in the difference Fourier map had negligible peaks. Interatomic distances obtained from the positional parameters of Table 1 are listed in Table 3. The Thermal Ellipsoid Plot computer program written by Johnson (1965) was used with Slater's (1964) radii to draw the stereoscopic view of the structure along the c axis. The illustration is presented in Fig. 1.

Discussion

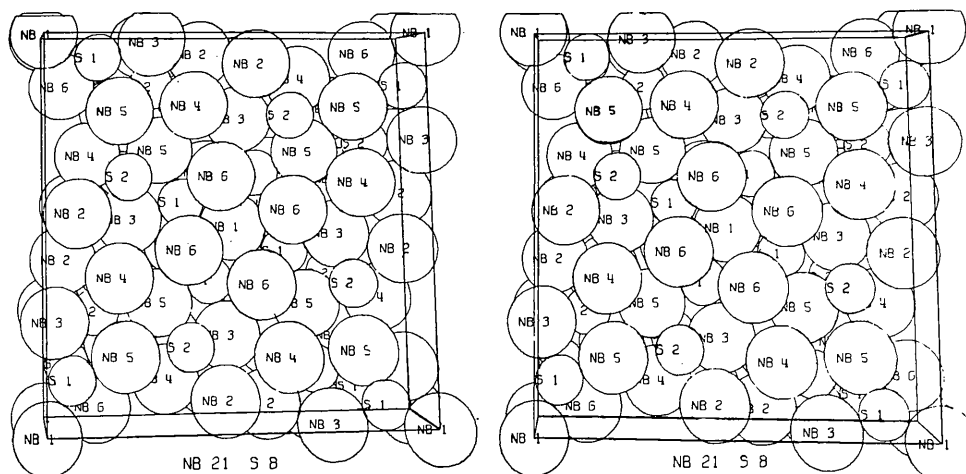
The apparent coordination of both independent sulfur atoms in Nb₂₁S₈ is similar to that of the sulfur atoms in Ti₂S (Owens, Conard & Franzen, 1967), to that of phosphorus and silicon in compounds with the cementite structure as discussed by Aronsson, Lundstrom & Rundqvist (1965) and to that of Ta and Nb in K₂TaF₇ and K₂NbF₇, respectively (Hoard, 1939). Each sulfur atom is surrounded by a slightly distorted trigonal prism of niobium atoms with additional close metal atoms around the waist of the trigonal prism, *i.e.* ideally, on lines perpendicular to the centers of the rectangular faces of the prism, as shown for S(1) in Fig. 2.

Table 1. Refined atomic positions for Nb₂₁S₈

| Wyckoff notation | x | y | $\sigma(x)$ | $\sigma(y)$ | B | $\sigma(B)$ |
|------------------|------|--------|-------------|-------------|---------------------|---------------------|
| Nb(1) | 2(a) | 0.0 | 0.0 | 0.0001 | 0.36 Å ² | 0.05 Å ² |
| Nb(2) | 8(h) | 0.0813 | 0.4502 | 0.0001 | 0.26 | 0.02 |
| Nb(3) | 8(h) | 0.2777 | 0.0202 | 0.0001 | 0.58 | 0.03 |
| Nb(4) | 8(h) | 0.3843 | 0.1851 | 0.0001 | 0.47 | 0.03 |
| Nb(5) | 8(h) | 0.1919 | 0.1986 | 0.0001 | 0.54 | 0.03 |
| Nb(6) | 8(h) | 0.4509 | 0.3643 | 0.0001 | 0.53 | 0.03 |
| S(1) | 8(h) | 0.1377 | 0.0637 | 0.0003 | 0.30 | 0.07 |
| S(2) | 8(h) | 0.2126 | 0.3644 | 0.0003 | 0.35 | 0.07 |

Table 2. Observed and calculated structure factors for Nb₂₁S₈

| h | k | l | Observed | Calculated | Observed | Calculated | Observed | Calculated |
|---|---|----|----------|------------|----------|------------|----------|------------|
| 1 | 0 | 0 | 31 | 31 | 81 | 81 | -68 | -68 |
| 1 | 0 | 1 | 33 | 33 | 145 | 145 | 25 | 160 |
| 1 | 0 | 2 | 2 | 2 | 50 | 50 | 158 | 158 |
| 1 | 0 | 3 | 2 | 2 | 50 | 50 | -167 | -167 |
| 1 | 0 | 4 | 4 | 4 | 70 | 70 | 29 | 29 |
| 1 | 0 | 5 | 4 | 4 | 70 | 70 | -97 | -97 |
| 1 | 0 | 6 | 2 | 2 | 50 | 50 | 68 | 68 |
| 1 | 0 | 7 | 2 | 2 | 50 | 50 | -35 | -35 |
| 1 | 0 | 8 | 2 | 2 | 50 | 50 | 68 | 68 |
| 1 | 0 | 9 | 2 | 2 | 50 | 50 | -114 | -114 |
| 1 | 0 | 10 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 11 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 12 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 13 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 14 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 15 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 16 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 17 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 18 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 19 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 20 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 21 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 22 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 23 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 24 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 25 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 26 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 27 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 28 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 29 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 30 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 31 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 32 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 33 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 34 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 35 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 36 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 37 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 38 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 39 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 40 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 41 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 42 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 43 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 44 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 45 | 2 | 2 | 50 | 50 | -64 | -64 |
| 1 | 0 | 46 | 2 | 2 | 50 | 50 | 103 | 103 |
| 1 | 0 | 47 | 2 | 2 | 50 | 50 | -64 | -64 |

Fig. 1. Stereoscopic illustration of the Nb_{21}S_8 unit cell viewed along the c axis.Table 3. Interatomic distances in Nb_{21}S_8 ($\pm 0.003 \text{ \AA}$)

| Reference atom | Neighbor | Number of neighbors | Distance |
|----------------|----------|---------------------|--------------------|
| Nb(1) | S(1) | 4 | 2.547 \AA |
| | Nb(6) | 8 | 2.949 |
| Nb(2) | S(2) | 1 | 2.634 |
| | Nb(2) | 4 | 2.819 |
| | Nb(4) | 2 | 2.884 |
| | Nb(3) | 2 | 2.946 |
| | Nb(2) | 1 | 3.202 |
| Nb(3) | S(1) | 1 | 2.462 |
| | S(2) | 2 | 2.570 |
| | Nb(4) | 2 | 2.902 |
| | Nb(2) | 2 | 2.946 |
| | Nb(6) | 2 | 3.140 |
| Nb(4) | S(2) | 2 | 2.482 |
| | Nb(5) | 2 | 2.877 |
| | Nb(2) | 2 | 2.884 |
| | Nb(3) | 2 | 2.902 |
| | Nb(6) | 1 | 3.210 |
| | Nb(5) | 1 | 3.238 |
| Nb(5) | S(1) | 1 | 2.440 |
| | S(2) | 2 | 2.551 |
| | S(2) | 1 | 2.806 |
| | Nb(4) | 2 | 2.877 |
| | Nb(5) | 2 | 3.100 |
| | Nb(6) | 2 | 3.112 |
| | Nb(4) | 1 | 3.238 |
| | | | |
| Nb(6) | S(1) | 2 | 2.533 |
| | S(1) | 2 | 2.548 |
| | Nb(1) | 2 | 2.949 |
| | Nb(5) | 2 | 3.112 |
| | Nb(3) | 2 | 3.140 |
| | Nb(4) | 1 | 3.210 |
| | | | |
| S(1) | Nb(5) | 1 | 2.440 |
| | Nb(3) | 1 | 2.462 |
| | Nb(6) | 2 | 2.533 |
| | Nb(1) | 1 | 2.547 |
| | Nb(6) | 2 | 2.548 |
| S(2) | Nb(4) | 2 | 2.482 |
| | Nb(5) | 2 | 2.551 |
| | Nb(3) | 2 | 2.570 |
| | Nb(2) | 1 | 2.634 |
| | Nb(5) | 1 | 2.806 |

The two bonding interpretations proposed for S in Ti_2S can be discussed with regard to Nb_{21}S_8 . The first interpretation was based on the assumption that all bond orders are given approximately by the empirical relationship due to Pauling (1960):

$$D(n) = D(1) - 0.6 \log n. \quad (1)$$

This interpretation yields, in this case, the approximate niobium and sulfur valences listed in column two of Table 4. These valences are reasonable if it is assumed that niobium has a maximum valence of five and that sulfur has a valence of about four, in accordance with

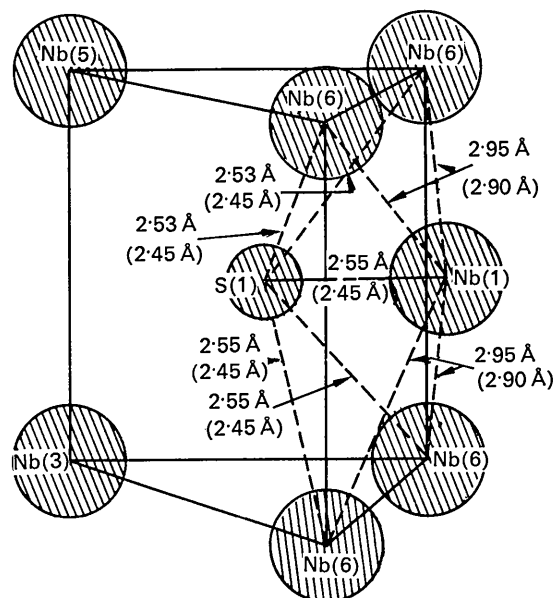


Fig. 2. A sulfur atom is surrounded by a slightly distorted trigonal prism of Nb atoms with an additional close metal atom just removed from one of the rectangular faces of the prism. The values in parentheses are summations of Slater's radii.

the bonding scheme utilizing sulfur *d* orbitals previously proposed (Franzen, 1966).

Table 4. Valences for Nb₂₁S₈

| | Valence based on 7-fold coordination | Valence based on trigonal prismatic coordination only |
|-------|--|--|
| Nb(1) | 4.97 | 2.86 |
| Nb(2) | 4.50 | 4.12 |
| Nb(3) | 3.62 | 3.62 |
| Nb(4) | 4.31 | 4.31 |
| Nb(5) | 3.86 | 3.67 |
| Nb(6) | 3.74 | 3.74 |
| S(1) | 4.22 | 3.69 |
| S(2) | 3.93 | 3.36 |

Interatomic distances less than 3.25 Å are used.

The second proposal was based on the hypothesis that sulfur bonding in compounds of this type is directional in nature, with the bonding orbitals directed toward the corners of a trigonal prism. The basis for this hypothesis was the marked tendency of sulfur, selenium and tellurium to form compounds in which the coordination of the chalcogen is trigonal prismatic and the lack of structures in which the chalcogens expanded their coordination beyond six. According to this interpretation, the short S(1)-Nb(1) distance would be primarily determined by the bonding of both S(1) and Nb(1) to four Nb(6), as shown in Fig. 2. The valences calculated assuming that the only niobium atoms to which the sulfur atoms are bonded are those at the corners of trigonal prisms but that equation (1) otherwise applies, are given in column three of Table 4 and are again in accord with the scheme in which sulfur *d* orbitals are utilized in the formation of sulfur-niobium bonds.

Evidence favoring the proposal that sulfur is bonded to 7 metals is provided by a comparison of the valences of Nb(1) in columns two and three of Table 4. The valence of 2.9 calculated on the assumption that the waist niobium atoms are not bonded to the sulfur is unreasonably low, while the value 5.0, obtained by the assumption of equation (1) only, is entirely reasonable for a compound of this kind.

It is concluded that the nature of the bonding of sulfur in compounds of the types discussed here (Ti₂S and Nb₂₁S₈) is metallic, *i.e.* that the bonding is of the delocalized, electron deficient type characteristic of metals. It is furthermore concluded that there is a significant contribution of the sulfur *d* orbitals to the valence (conduction) band.

The authors acknowledge the suggestion of Professor J. D. Dunitz that the direct method be used in the structure determination. Prior attempts at solving the Patterson function had been fruitless. The authors express their appreciation to Drs R. A. Jacobson and D. R. Fitzwater and their co-workers for assistance in using the IBM 7074 and 360 systems, to Mr Don Bailey for his assistance in taking intensity data, and to R. Z. Bachman and D. E. Carlson for performing combustion analyses.

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